

Name: Solutions

For #'s 1-9, solve each equation for all solutions (rational, irrational & imaginary)

1. Solve $x^2 + 49 = 0$

$$\begin{aligned} x^2 &= -49 \\ \sqrt{x^2} &= \pm \sqrt{-49} \\ x &= \pm i\sqrt{49} \end{aligned}$$

$$x = \pm i \cdot 7$$

$$\{7i, -7i\}$$

2. Solve $2y^2 - 64 = 0$

$$\begin{aligned} 2y^2 &= 64 \\ y^2 &= 32 \\ \sqrt{y^2} &= \pm \sqrt{32} \end{aligned}$$

$$y = \pm \sqrt{16 \cdot 2}$$

$$y = \pm 4\sqrt{2}$$

$$\{4\sqrt{2}, -4\sqrt{2}\}$$

3. Solve $p^2 - 4p + 2 = 0$

$$\begin{aligned} p &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} \\ p &= \frac{4 \pm \sqrt{16 - 8}}{2} \end{aligned}$$

$$p = \frac{4 \pm \sqrt{8}}{2}$$

$$p = \frac{4 \pm 2\sqrt{2}}{2}$$

$$\{2 + \sqrt{2}, 2 - \sqrt{2}\}$$

4. Solve $w^2 + 6w + 10 = 0$

$$\begin{aligned} w &= \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)} \\ w &= \frac{-6 \pm \sqrt{36 - 40}}{2} \end{aligned}$$

$$w = \frac{-6 \pm \sqrt{-4}}{2}$$

$$w = \frac{-6 \pm i\sqrt{4}}{2}$$

$$w = \frac{-6 \pm 2i}{2}$$

$$w = -3 \pm i$$

$$\{-3 + i, -3 - i\}$$

5. Solve $3t^3 + 36t = 0$

$$\begin{aligned} 3t(t^2 + 12) &= 0 \\ \downarrow & \quad \downarrow \\ t=0 & \quad t^2 + 12 = 0 \\ & \quad t^2 = -12 \end{aligned}$$

$$\sqrt{t^2} = \pm \sqrt{-12}$$

$$t = \pm i\sqrt{12}$$

$$t = \pm i\sqrt{4 \cdot 3}$$

$$t = \pm 2i\sqrt{3}$$

$$\{0, 2i\sqrt{3}, -2i\sqrt{3}\}$$

6. Solve $10y^3 + 20y^2 + 5y = 0$

$$\begin{aligned} 5y(2y^2 + 4y + 1) &= 0 \\ \downarrow & \quad \downarrow \\ y=0 & \quad y = \frac{-4 \pm \sqrt{4^2 - 4(2)(1)}}{2(2)} \\ & \quad y = \frac{-4 \pm \sqrt{16 - 8}}{4} \end{aligned}$$

$$y = \frac{-4 \pm \sqrt{8}}{4}$$

$$y = \frac{-4 \pm 2\sqrt{2}}{4}$$

$$y = \frac{-2 \pm \sqrt{2}}{2}$$

$$\left\{ \frac{-2 + \sqrt{2}}{2}, \frac{-2 - \sqrt{2}}{2} \right\}$$

7. Solve $x^3 - 2x^2 - 20x + 40 = 0$

$$x^2(x-2) - 20(x-2) = 0$$

$$(x-2)(x^2 - 20) = 0$$

$$\downarrow$$

$$x=2$$

$$\downarrow$$

$$x^2 - 20 = 0$$

$$x^2 = 20$$

$$x = \pm\sqrt{20}$$

$$x = \pm 2\sqrt{5}$$

$$\{2, 2\sqrt{5}, -2\sqrt{5}\}$$

8. Solve $p^4 - 5p^2 - 36 = 0$

$$(p^2 - 9)(p^2 + 4) = 0$$

$$(p-3)(p+3)(p^2 + 4) = 0$$

$$\downarrow$$

$$p=3$$

$$\downarrow$$

$$p=-3$$

$$\downarrow$$

$$p^2 + 4 = 0$$

$$p^2 = -4$$

$$p = \pm\sqrt{-4}$$

$$p = \pm i\sqrt{4}$$

$$p = \pm 2i$$

$$\{3, -3, 2i, -2i\}$$

9. Write an equation in standard form for the degree-2 polynomial function with the given solution set $\{2i, -2i\}$

$$(x-2i)(x+2i) = 0$$

$$x^2 + 4 = 0$$

$$x^2 + 2ix - 2ix - 4i^2 = 0$$

$$x^2 - 4(-1) = 0$$

10. Write an equation in standard form for the degree-2 polynomial function with the given solution set $\{\sqrt{13}, -\sqrt{13}\}$

$$(x-\sqrt{13})(x+\sqrt{13}) = 0$$

$$x^2 + \sqrt{13}x - \sqrt{13}x - \sqrt{169} = 0$$

$$x^2 - 13 = 0$$

11. The solution set is $\{1, i\sqrt{6}, -i\sqrt{6}\}$, and the polynomial has degree 3

$$(x-1)(x-i\sqrt{6})(x+i\sqrt{6}) = 0$$

$$(x-1)(x^2 + i\sqrt{6}x - i\sqrt{6}x - i^2\sqrt{36}) = 0$$

$$(x-1)(x^2 - (-1)(6)) = 0$$

$$(x-1)(x^2 + 6) = 0$$

$$x^3 - x^2 + 6x - 6 = 0$$

Optional Extra Credit

Solve $2x^3 - 8x^2 + 5x + 3 = 0$ for all three solutions given that $x-3$ is one factor of the polynomial.

$$\begin{array}{r} 3 \overline{) 2 \ -8 \ 5 \ 3} \\ \underline{6 \ -6 \ -3} \\ 2 \ -2 \ -1 \ 0 = R \\ 2x^2 - 2x - 1 = 0 \end{array}$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{2 \pm \sqrt{4+8}}{4}$$

$$x = \frac{2 \pm \sqrt{12}}{4}$$

$$x = \frac{2 \pm 2\sqrt{3}}{4}$$

$$x = \frac{1 \pm \sqrt{3}}{2}$$

$$\left\{3, \frac{1+\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}\right\}$$